Highlighted in this PDF are Particular Pages of Interest for the 2017 Georgia FFA CDE ENR Team Activity



Basic Construction Surveying

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TABLE OF CONTENTS

Chapter 1: Survey Stakes

Chapter 2: Measurement of Distances

Chapter 3: Slope Measurements

Chapter 4: Common Mistakes in Reading and Recording Linear Measurements

Chapter 5: Leveling

Chapter 6: Notekeeping

1-1. a. Survey stakes

1-2. c. Hubs

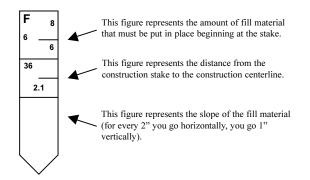
1-3. d. 50 foot

CHAPTER 1: SURVEY STAKES

Survey stakes are the survey party's means of communication. The survey stakes most frequently used by the Department of Transportation are the wooden 1" x 2" x 18" stakes, although other sizes are available. Survey stakes are normally set by a survey party to establish and designate a work point or reference point on the ground. Appropriate information is written on the stake with keel, felt-tip markers, or other devices, instructing construction personnel as to the work to be performed. Since these stakes are the actual physical basis for construction, it is essential that they be set at the correct location with the data written on them in a legible and understandable manner.

2" x 2" wooden stakes, called "hubs," are available to mark important survey points, work points, or reference points, which are to remain in place for future use. These hubs are normally driven flush with the ground after which a survey tack is set in the top of the hub to mark the exact survey point. The hub is driven flush to make it less susceptible to damage and its location is usually marked by a "guard" or "witness" stake. The guard stake will usually have sufficient information written on it to identify the point.

FRONT OF STAKE



BACK OF STAKE

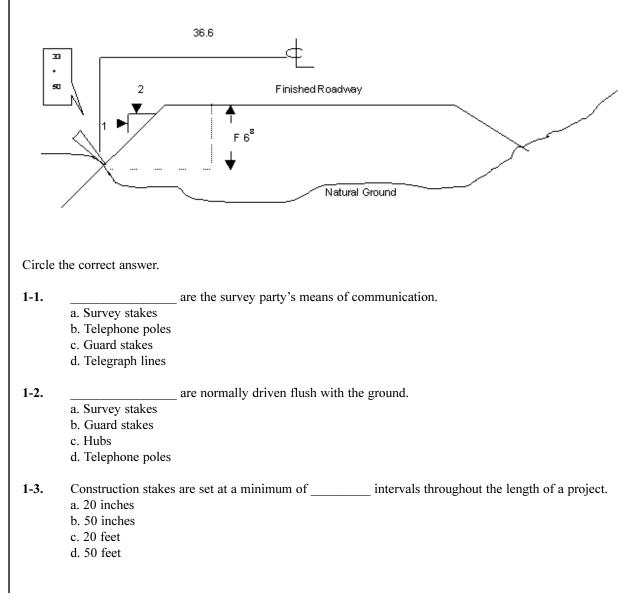
33

+

50

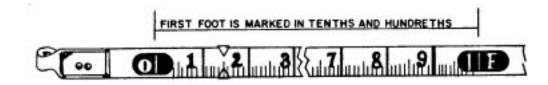
This figure represents a station, the distance that this particular stake is from the beginning of the project (EX: 3350' from beginning of project).

Construction stakes are set at a minimum of 50' intervals throughout the length of a project. They are the means of communication between the surveyor and the contractor.



CHAPTER 2: MEASUREMENT OF DISTANCES

Measurements are usually made with a steel tape called a chain; however, a cloth tape may be used at times for measuring distances when accuracy is not critical. The steel chains used in construction surveying are normally 100' long, graduated in 1-foot increments, except for the first and last foot, which is graduated in tenths and hundredths of a foot.



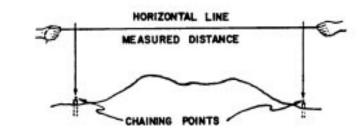
TYPICAL GRADUATIONS ON A STEEL CHAIN

The steel chains used in construction surveying are commonly called "cut chains." This terminology is used because in order to obtain a partial distance other than a whole foot, the front chain person must "cut" some portion of the first foot in order to read a true distance. One can readily see that the distance from the zero (0) on the chain to the 64-foot mark is 64 feet. Now assume that the back chain person held the 64-foot mark on a point and the front chain person read 4 marks to the right of the one-tenth mark, or 0.14 feet. This would mean that the measured distance is 64 feet minus 0.14 feet (64.00' - 0.14') or 63.86 feet. By the same token, if the back chain person held 64 feet on a point and the front chain person cut 0.94 feet on another point, the distance between the two points would be 63.06 feet (64.00' - 0.94'= 63.06'). Normally, the back chain person would call out the distance he is holding ("Holding 64 feet") and the front chain person would call out the distance he is cutting ("Cutting 94 hundredths"). Both chainmen would mentally compute the distance, one would call it out (63.06 feet") and the other would acknowledge its correctness. Both chainmen should always be mindful of the distances being measured, particularly long distances where lack of attention could result in a measurement being recorded one or more chain lengths in error.

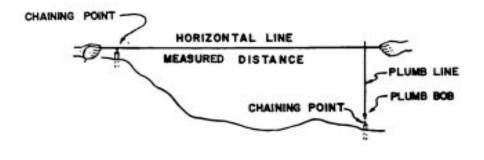
MEASUREMENT OF HORIZONTAL DISTANCES

In surveying, the distance between two points means the horizontal, or level, distance. Construction plans are prepared showing distances as horizontal. Two methods are commonly used to measure the horizontal distance between two points.

The first method is to measure the distance by holding the tape in a horizontal position. This method provides a direct determination of the distance.



The second method, slope chaining, will be discussed later in the text. Unless the two chaining points are at the same elevation, this method also involves the use of plumb bobs to transfer the distance to the ground, or chaining points.

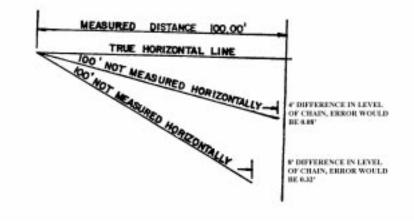


A plumb bob is a device usually made of brass with replaceable steel tip and attached to a special cord, which may be used to transfer a point vertically. It is commonly used in measuring distances and in alignment operations. The customary way to hold the plumb bob when providing a vertical line of sight is to let the string drape across the forefinger and then clamp the string with the thumb.



The importance of having both ends of the chain at the same elevation when measuring a horizontal distance cannot be over emphasized. One of the most common errors in chaining is failure to hold both ends of the chain at the same level.

Holding a chain level requires practice. A good chain person will use guides such as finish lines on buildings and water surfaces in leveling the chain. Also, he will readily use a hand level if other reliable guides are not available. The drawing below shows how errors in determining horizontal distances can occur as a result of failure to keep both ends of the chain level.

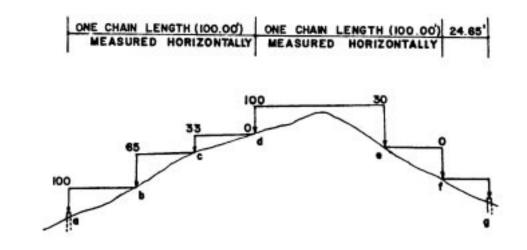


When chaining by the usual method of holding the chain horizontally, and steep or irregular terrain is encountered, it may be necessary to measure a distance by series of individual horizontal measurements, each less than a chain length.

The length of each horizontal increment would be controlled by the slope of the terrain; steeper terrain would naturally result in shorter chaining increments. One method of measuring steep terrain would be to lay the chain out for its entire length along the line to be measured, then the back chain person would hold the 100-foot mark over the beginning point, Point "a", at a convenient elevation using a plumb bob. The front chain person would then take position at a point, Point "b", along the line so that the chain could be held horizontally and mark the distance on the ground at some full foot mark (for instance, at the 65-foot mark).

The process would then be repeated by having the back chain person come ahead to Point "b" and hold the same 65-foot mark on the chain over Point 'b" at some convenient elevation. The front chain person would then progress ahead to a point where the chain could be held horizontally near the ground, Point "c", and again mark the distance on the ground at some full foot mark (for instance, at the 33-foot mark).

The process would be repeated as necessary until the front chain person can mark a point at the zero (0) end of the chain, Point "D". At this point one chain length (100.00 feet) would have been measured. The entire process would then be repeated for as many chain lengths as necessary to measure the entire distance. Measurement of distances in the above manner is called "breaking the chain" or "breaking tape".



You will note that the distance to be measured was over two full chain lengths. In that particular example, the two full lengths were measured by "breaking the chain" as necessary to traverse the terrain. Then with the back chain person on Point "f" holding 25 feet, the front chain person read, or cut, 0.35' to complete the measurement.

You will recall from previous discussions in this text that in this example, the back chain person would call out "holding 25 feet," the front chain person would call out "cutting 35 hundredths," and both would mentally compute the distance (25.00' - 0.35' = 24.65'), with one chain person calling out "24.65 feet," and the other chain person verifying this figure. Then the two chain lengths (200.00 feet) could readily be added to the partial chin length of 24.65 feet for the total measured distance of 224.65 feet. The big advantage of "breaking the chain" is that it is easy to keep account of the number of full chain lengths, 100.00 feet, measured.

One can readily see that the distances could have been measured individually (a to b = 35 feet, b to c = 32 feet, c to d = 33 feet, etc) and then totaled for the overall horizontal distance; however, this would require additional computations whereas "breaking the chain" lets you utilize the chain itself to sum the partial distances necessary to make up a chain length. As previously noted, "breaking the chain" has the advantage of working in even chain lengths (100 feet) in lieu of odd distances.

The above text discussed in detail horizontal measurements on uphill terrain; however, the same principles apply for horizontal measurements of a downslope. The only variation would be that the back chain person would be working at or near ground elevation and the front chain person would be doing the plumbing.

Horizontal measurement with the chain and plumb bobs is a process which requires skill and care. It takes considerable practice and experience to chain accurately and efficiently. It is easier to chain downhill than uphill.

When chaining downhill, the back chain person can hold the tape steady at the ground level point, while the front chain person applies the required tension or pull to the chain, and then plumbs exactly over the pint to be measured.

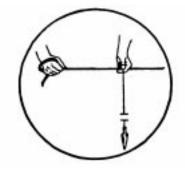
The front chain person is applying the tension and doing the plumbing, thus it is easier for him to maintain his balance. The back chain person has no balance or plumbing problem since he is working at ground level and can position himself solidly while holding the chain over the point. When chaining uphill, the back chain person must keep himself in balance and hold his plumb bob exactly and steadily over the pint while the front chain person applies the required tension. This tends to pull the back chain person off balance.

SUGGESTED CHAINING PROCEDURES

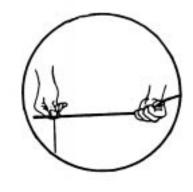
When chaining, both chainmen should position themselves so that they can gradually apply the required tension on the chain without losing their balance. One should never apply a sudden or irregular pull on the chain while attempting to make a measurement; to do so, could cause the other chain person to lose his balance. The chain

person should always stand so that he can look squarely across the graduations on the chain when reading a measurement.

When using a plumb bob, the chain person should get a firm grip on the chain with one hand, and manipulate the plumb bob with the other. Normally one should not use the hand controlling the plumb bob to apply tension to the chain. The leather throngs on the chain ends can be wrapped around the chain person's hand to provide a steady grip to apply tension in most instances.



However, when "breaking the chain" or measuring distances less than a chain length, it will be necessary to grip the chain itself.



Care should be exercised so as not to permanently bend or "kink" the chain when gripping it. Generally the back chain person can drape the plumb bob string across the chain on the appropriate foot mark, and clamp the string position with the thumb and forefinger of one hand while applying tension to the chain with the other. Care should

2-6. should not

2-7.

This terminology is used because partial measurements, i.e. less than a whole foot, must often be made. The front chain person must "cut" some portion of the first foot in order to read a true distance.

2-8. to transfer a point vertically

be given to ensure that the plumb bob string remains in the correct position on the chain during the measurement process. When using the plumb bob to make a measurement, the plumb bob tip should be held approximately 1/8 of an inch above the chaining point; if held much higher, it is difficult to position the tip directly over the chaining point.

In general, the chain person should not try to plumb higher than his chest because it is very difficult to maintain balance and provide a steady pull on the chain above chest level. The chain person should be careful to stand far enough off line to allow line of sight for the transit person.

When checking measured distances, the front and back chain person should exchange positions, or if possible, two other chain person should be used. Also, when checking measurements greater than one chain length, different intermediate chaining points should be used.

Answer the questions.

2-1. A cloth tape may be used in surveying when ______. a. accuracy is not important b. accuracy is important c. when it's full length only d. when it's half length only

- 2-2. Steel chains are commonly called ______ in construction. a. steel rivets b. cut stakes c. cut chains d. measuring chains
- **2-3.** If the front chain person reads three marks to the right of the two-tenths mark and the back chain person has his point marked at 75, what is the distance?
- **2-4.** If the back chain person calls "Holding 52 feet,." and the front chain person calls "cutting 37-hundreths (.37)," what is the distance they both should acknowledge?
- **2-5.** A plumb is used to:
 - a. transfer grades to a hub
 - b. transfer grades to a stake
 - c. transfer a point horizontally
 - d. transfer a point vertically

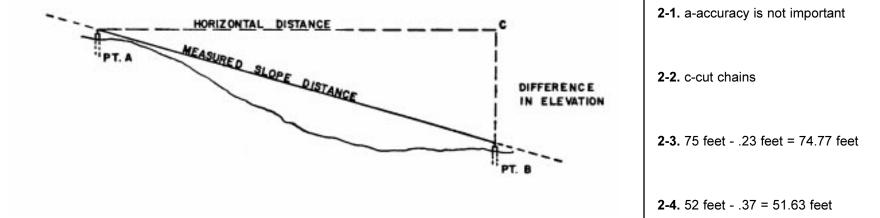
- **2-6.** The chain person (should/should not) plumb higher than his chest.
- **2-7.** Why are the chains commonly used in highway construction surveying called "cut chains?"
- **2-8.** Why are plumb bob's used when chaining?

CHAPTER 3: SLOPE MEASUREMENTS

In some instances it may be easier and more accurate to make a slope measurement rather than a horizontal measurement. The cases are usually confined to distances of a chain length or less and on steep terrain.

There are two common methods of computing the horizontal distance from a slope measurement.

The first method is by measuring a slope distance directly between two points when the difference in elevation of the two points is known.



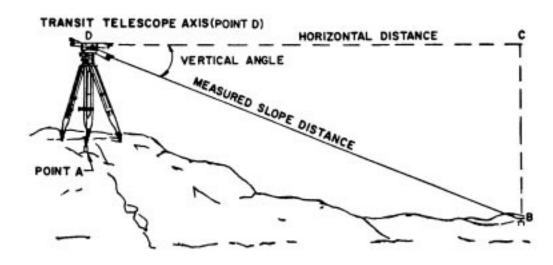
You can see that the horizontal distance can be readily computed based on the right triangle rule, that the square of the hypotenuse is equal to the sum of the squares of the other two sides. It is assumed that the reader is familiar with this rule from the Basic Highway Mathematics Course, or from some other source, and development of this rule will not be discussed.

Suppose one needs to determine the horizontal distance between Points A and B, and it is determined that the difference in elevation between Point A and Point B is 8.26 feet. Then using a chain the distance between Point A and Point B is measured to be 73.84 feet. This is the slope distance of Line AB. Remember the measurement is a direct measurement; it is not a horizontal measurement. You can see that a right triangle has been constructed with Line AB as the hypotenuse. Recalling the rule mentioned above, it can be seen that AB2=AC2 + BC2 or solving this equation for the horizontal distance AC2 = AB2-BC2. Substituting in the above numbers, AC2 = (73.84)2 - (8.26)2 = 5452.35 - 68.23 = 5384.12. Therefore Line AC = 5384.12, AC = 73.38 feet.

The most frequently used method of slope measurement in construction surveying involves the use of the transit to

2-5. c-transfer a point horizontally

determine the angle between a horizontal line of sight at the transit axis, and a slope line from the transit axis to the point in question.



This angle is referred to as the "vertical angle." The transit will be discussed later on in this text, but for purposes of this article, it will be sufficient to say that the transit is an engineering instrument capable of measuring both horizontal and vertical angles.

The transit is set up over one point, and the vertical angle is measured to the other point. It should be noted that this method will work equally well with the transit located on the lower point. The distance is then measured directly from the transit telescope axis to the point in question.

One can see that a right triangle has been constructed with the measured slope distance, Line DB, as the hypotenuse. Recalling a basic trigonometric function, one can see that by knowing the vertical angle and the length of the hypotenuse, the horizontal distance can be computed by use of the Cosine function of a right triangle; Cosine of the angle equals the adjacent side divided by the hypotenuse.

In the right triangle BCD formed, the cosine of the vertical angle equals Line DC/Line BD. As an example, assume that the transit was set up over Point A, the vertical angle was measured to be 7° - 00', and the slope distance DB was measured to be 81.84 feet. Then the horizontal distance DC could be computed as follows: Cosine 7° - 00' = DC DC = 81.84 X Cosine 7° - 00' = 81.84 X .99255 = 81.23

When using the transit in conjunction with slope measurements, the chain person should exercise care not to

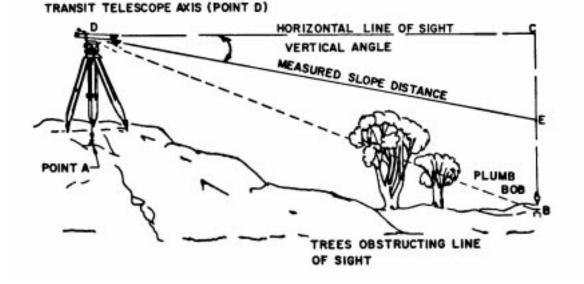
3-1. 1-Determine the elevation of various points 2-determine distances between points

3-2. may

3-3. can

disturb the instrument when chaining from the telescope axis. The chain person should also remember to base his measurement from the telescope axis, not the point of the ground, since the vertical angle is measured about the telescope axis.

One variation of the above text covering slope measurements should be mentioned here. On occasion it may be desirable to determine a horizontal distance by using the vertical angle and slope measurement when an obstruction blocks the instrument person's direct line of sight to the other point.



By use of a plumb bob, the front chain person can transfer the point on the ground (Point B) vertically to some convenient elevation (Point E) so that the instrument person has an unobstructed view. Then a new vertical angle and slope distance (Line DE) can be measured. Using the principles discussed earlier in this article, one could readily compute the horizontal distance (Line DC). For example, assume the vertical angle to be $4^{\circ} - 15'$ and the measured slope distance DB to be 81.45 feet. The horizontal distance DC is equal to the cosine of $4^{\circ} - 15'$ multiplied by 81.45 feet, or 81.23 feet.

Care should be taken to ensure that the front chain person holds the chain and plumb bob at the exact line of sight as determined by the vertical angle. This is extremely critical. For as one can see from the above examples, the measured slope distances for the same horizontal line vary significantly as the slope angle changes. The amount of error in computing a horizontal distance which results from failure to hold the chain at the exact line determined by the vertical angle drastically increases as the size of the vertical angle increases

Answer the questions.

3-1.	What are two common methods of measuring slopes, other than by "breaking chain?" 1 2
3-2.	It (may/may not) be easier and more accurate to make a slope measurement rather than a horizontal measurement.
3-3.	Horizontal distance (can/cannot) be readily computed based on the right triangle rule.

CHAPTER 4: COMMON MISTAKES IN READING AND RECORDING LINEAR MEASUREMENTS

Reading the Wrong Foot Mark

This is a careless mistake, which can usually be avoided by alertness and concentration by the chain person.

Transposing Figures

Sometimes a chain person may mentally transpose figures before calling them out, or the notekeeper may transpose them when recording them. For example, he may call out, or record 42.63 when he should have called out 42.36. Alertness and concentration will usually eliminate this type of mistake.

Reading the Chain Upside Down

When reading a chain upside down, certain numbers can easily be misread for others. For example, a 6 may be misread as a 9, 86 may be misread for 98, etc. One way to avoid this mistake is to note the adjacent footmarks on both sides of the footmark being used to see that the number is in sequence. However, the best solution is to avoid reading the chain in an upside down position.

Subtracting incorrectly when the front chain person cuts a partial foot.

This mistake can be minimized by both chainmen making the subtraction mentally in order to check each other. In order to do this, both chainmen must call out their readings for the measurement.

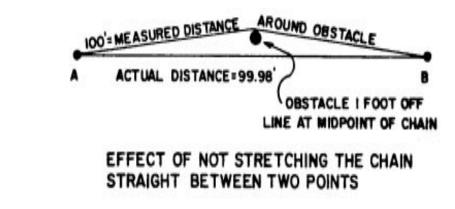
Principal Sources of Error in Linear Measurements

An error may be defined as the difference between an observed value and the true; or, as the variation in observations, measurements, or calculations, of a quantity. This section will discuss some of the principal errors made in chaining and their effect on the measurements. In general, these errors tend to result in the recorded, or measured, distances being greater than the actual distance.

Tape Not Stretched Straight

Chainmen should always be sure that the chain is free to be stretched directly between the two chaining points. The chain should not be allowed to bend around trees, bushes, boulders, etc., since this would result in the

measured distance being too great.



One can readily see the effect of this error on the measured distance as compared to the actual distance.

The wind can blow the chain off line, particularly when plumbing, resulting in the same type error as an obstacle which prevents the chain from being stretched straight between two points. The magnitude of this type error can be reduced by supporting the chain on line at one or more intermediate points.

Incorrect Alignment

When measuring distances greater than a chain length, or when breaking the chain, the intermediate chaining points must be lined by eye or by an instrument. Usually, the back chain person can stand on line and line the front chain person to within 0.2 feet to 0.3 feet of true line.

MEASURED DISTANCE= 100.00	MEASURED DISTANC	TRUE DISTANCE E=100.00 99.995	TRUE DISTANCE 11.958
99.999 TRUE DISTANCE	99.999 TRUE DISTANCE	MEASURED 10 DISTANCE	ME 12.00 MEASURED DISTANCE
EXAMPLE OF ER DISTANCES RESU ALIGNMENT OF 1	TING FROM IMPR		~~~~

One can see that if the chain is lined within 0.5 foot of true line, the error is insignificant when measuring a full chain length. This error may be computed by utilizing the right triangle rule as previously discussed. One can also see that for shorter distances, the alignment becomes more critical. Since this type error is cumulative, it becomes more significant when multiple chaining points are used in measuring a distance. This type error results in the measured distance being greater than the actual distance.

Careless Plumbing

This error relates to failure to hold the plumb bob tip directly over the chaining point. This type error is a compensating error in that the measured distance may be greater, or it may be less than the actual distance. Proficient use of the plumb bob requires practice and care. The chainmen should exercise care to ensure that the plumb bob is positioned directly over the chaining point when making a measurement.

Incorrect Length of Chain

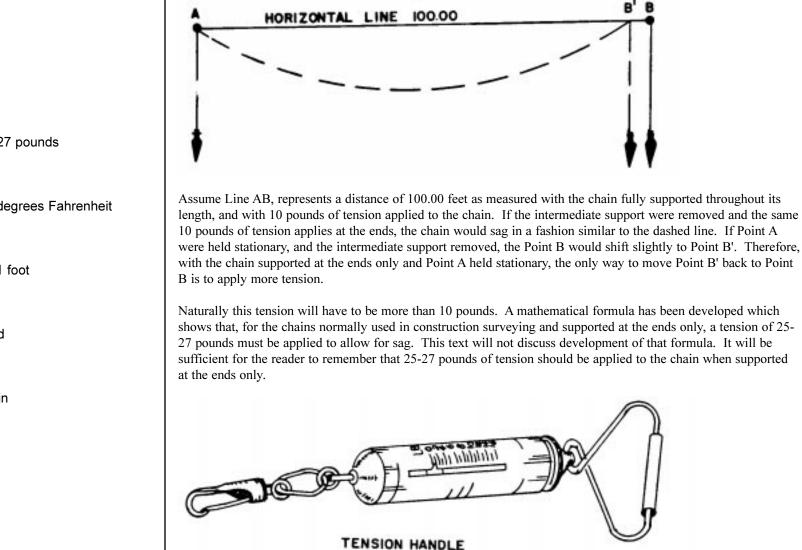
Obviously, if a chain is incorrectly marked, it cannot be used to accurately measure distances, unless it is recalibrated. For purposes of this course, the reader can assume that the chains used in construction surveying are manufactured to acceptable tolerances and therefore not be concerned with this type error. However, the reader should be aware that this error can exist.

Temperature

The reader should be aware that since chains are manufactured from steel, they are subject to expand and contract with variations in temperature. Chains are manufactured to be a standard length at 68° F. A good rule of thumb is that for each 15° F change in temperature, the corresponding change in length of a 100 foot chain will be approximately 0.01 foot.

Tension and Sag

Chains of 100 feet or less are manufactured to be of standard length when supported throughout the entire length on a level surface with 10 pounds of tension applied to its ends. However, this ideal condition is not frequently encountered in actual construction. Normally measurements will be made over irregular surfaces, and in most instances, the chain will be supported at its ends only. Obviously when the chain is supported at its ends only, it will sag.



4-2. a-25-27 pounds

4-3. a-63 degrees Fahrenheit

4-4. a-0.01 foot

4-5. should

4-6. c-chain

4-7. c-10

Most construction offices have a tension handle available for the more precise measurements. The tension handle is a spring balance device used to measure the tension applied to a chain. One end connects to the end of the chain, the other end is a handle for gripping and applying tension, and the center section is a spring-loaded slide graduated in pounds for registering the applied tension directly. The tension handle is not normally used in the day to day operations of a survey party, but the chain person should use the tension handle enough to keep a feel for the tension normally required when chaining. Even though the chain is constructed from steel, it has some elastic properties, and stretches when tension is applied. If the tension is greater or less than the standardized pull, the chain will be correspondingly too long or too short.

Care of the Steel Chain

The chain should be cleaned as necessary at the end of each day's operations to prevent rust. Damp or muddy chains should be cleaned, dried, and wiped with an oiled cloth before storing. An occasional wiping with a slightly oiled cloth is usually sufficient to prevent rust on chains that are only dusty. If some rust spots do accumulate, they can usually be removed by rubbing dry cement on the affected areas.

Care should be taken to prevent bending or kinking the chain. Never bend the chain sharply because this normally leaves a permanent kink in the chain. Never loop the chain around a survey stake or small sapling when dragging the chain around a sharp bend. It is best to let the other chain person stand at the point and feed the chain through his hands to prevent a sharp bend in the chain.

The front chain person should always look back to see that the chain is clear before pulling ahead for the next measurement; sometimes, when the chain is carelessly laid down, it will form a loop which if pulled tight will cause a kink or possibly a break. Any time a kink is created, it makes the chain more susceptible to breakage at that point.

Vehicles should not be allowed to run over the chain. Rubber tired vehicles can bend the chain, and, obviously, track type vehicles will severely damage a chain.

Answer the questions.

- 4-1. Name four of the principal sources of error in linear measurements?
 - 1.

 2.

 3.

 4.

4-2.	 The standardized tension, which must be applied to a steel chain when only supported by the ends is a. 25-27 pounds b. 10-12 pounds c. enough to hold up a 200 lb. person d. 100 pond weights
4-3.	 The temperature standard at which chains are manufactured is a. 63 degrees Fahrenheit b. 32 degrees Fahrenheit c. 15 degrees Fahrenheit d. 63 degrees Celsius
4-4.	 For each 15 degrees F. change in temperature, the chain change in length, approximately a. 0.01 foot b. 0.001 foot c. 1.0 foot d. 10.0 foot
4-5.	A chain (should/should not) be cleaned at the end of the day's operation.
4-6.	A tension handle is used to measure tension applied to a a. level b. transit c. chain d. stake
4-7.	The tension to be applied to the chains normally used in construction surveying when the chain is supported throughout the entire length is a. 150 b. 15 c. 10 d. 100

CHAPTER 5: LEVELING

Leveling may be defined as the process of determining the elevations of various points, or determining the vertical distances between points. An elevation may be defined as the vertical distance above a reference plane, or datum. Mean sea level is the datum most commonly used in leveling; however, in some cases, it may be advantageous to assume, or set, a datum. Various organizations such as the National Geodetic Survey, the U. S. Geological Survey, and the DOT's Office of Location/ Control Surveys Party have conveniently established a network of monuments, called "benchmarks," which are referenced to mean sea level. These benchmarks are the references normally used to establish the vertical control on highway construction projects.

In the process of accumulating data for the development of a set of roadway plans, the Photogrammetry Unit and/or the Location Unit will establish benchmarks along the project at frequent intervals for use during construction of the project. These benchmarks are usually nails and spikes driven in tree trunks, or X-marks etched into some hard and permanent surface such as concrete. The benchmarks are appropriately described on the construction plans with the elevations designated.

The Engineer's Level

The engineer's level is an instrument designed to project a level line of sight. Basically, the engineer's level consists of a telescope tube designed to impose a set of cross-hairs on the image viewed through the telescope. A spirit level vial is attached or incorporated into the instrument to allow the tube to be accurately leveled by manipulating adjusting screws on a foot plate.



tape not stretched straight

incorrect alignment

careless plumbing

incorrect length of chain

temperature

tension and sag



There are three types of levels used in construction surveying: The Wye Level, the Dumpy Level, and the Self-Leveling Level. All are designed to project a level line of sight when used properly.

The Wye Level

The Wye Level is basically a telescope which is attached to a horizontal bar by two vertical supports. A sensitive spirit level is attached parallel to the telescope for use in leveling the instrument. The horizontal bar is parallel to the telescope and is mounted on a vertical tapered spindle, which fits down into a conical socket in the leveling head. The leveling head is attached to the foot plate by a ball and socket type connection to allow for movement when the leveling screws are turned. The foot plate is threaded to screw onto the tripod head. The characteristic, which distinguishes the Wye level, is the manner in which the telescope is secured to the horizontal bar. The telescope is clamped in two Y shaped supports, which are easily opened, so that the telescope may be reversed end-for-end when the instrument is being adjusted. The Y supports are attached to the horizontal bar with capstan nuts and at least one Y is adjustable vertically.

The Dumpy Level

The Dumpy Level is constructed similar to the Wye type level. The basic difference between the, two types of instruments is that the telescope is rigidly attached to the horizontal bar on the Dumpy Level, and thus this type of instrument cannot be adjusted in the same manner as a Wye Level. However, the cross hair ring and the spirit level tube may be adjusted on the Dumpy Level.

The Self-Leveling Level

The Self Leveling type uses an optical system to establish a level line of sight, instead of the spirit level bubble used in the Wye and Dumpy levels. The optical system consists of three prisms within the telescope, which reflect the line of sight. The two end prisms are stationary, while the center prism is suspended from the top of the telescope by wires and is free to swing under the force of gravity. The optical system is designed so that a level line of sight is maintained as long as the center prism hangs freely, even though the telescope tube may not be truly level.



The Hand Level

The Hand Level is a device commonly used in construction when a great degree of accuracy is not required. The Hand Level is often referred to as a "Locke" Level. An experienced observer can get satisfactory results with the Hand Level when the distances involved are approximately 50 feet or less. At distances greater than 50 feet, readings with the Hand Level may be questionable.

The Hand Level is a metal tube with a spirit level tube attached to its top. The metal tube is slotted under the spirit level and a cross-wire is positioned across the slot. There is a prism inside the metal tube, which reflects the cross-wire and level bubble when the observer looks into the tube. By tilting the farther end of the instrument up or down the bubble can be made to appear bisected by the cross-wire. When this occurs, the cross-wire, represents a level line of sight.

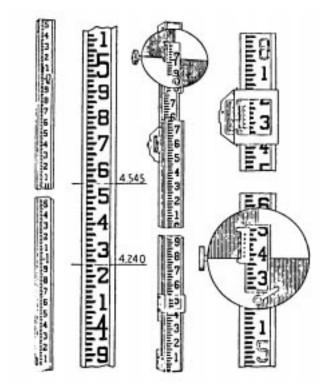
The field of view through a Hand Level is split. The left half shows the level bubble and cross-wire, while the right half provides the user an open view of the landscape, level rod, or other objects. The user should remember that the level bubble must be on the top side of the tube for proper operation. The cross-wire which, defines the level line can usually be adjusted by means of two opposing screws.



Leveling Rods

Level rods are normally constructed from strips of wood and graduated in hundredths of feet. The rod most commonly used in construction surveying is called the "Philadelphia" rod. The student should be aware that other-types of level rods are made, but they will not be discussed in this text. Level rods are available in several lengths;

however, the one commonly used in construction surveying is a three-piece device, which is twelve feet long-when fully extended. The three sections of the rod are assembled to slide one over the other, and when fully extended, the sections are locked in place by screw clamps.



The above photo shows several views of the Philadelphia level rod. The target shown on the two right hand views is not normally used in construction surveying. The face of the rod is painted white with the graduations painted in black and spaced 0.01 ft. apart. The top of the black graduations are the even hundredths, and thus the top of the white graduations are the odd hundredths. The smaller numbers painted black represent tenths of a foot; the large numbers painted red indicate foot-marks.

Setting Up The Level

When setting up the level to take a reading, the tripod legs should be positioned at such an angle as to provide stability and a convenient working height. The tripod legs should also be in a position, which leaves the tripod

head in a nearly horizontal position; this will facilitate the leveling of the instrument with its leveling screws. The student should practice setting up a tripod without the instrument attached until he gets a fell for the procedure.



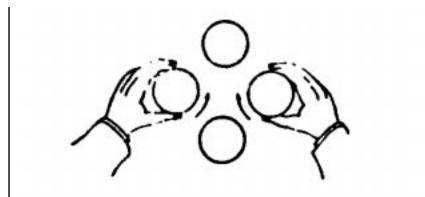
An accepted method of setting up the level is to place one tripod leg on the ground while holding the other two as shown in the above photo, and by using a fashion as to make the tripod head nearly horizontal. The tripod legs should be pressed firmly into the ground for added stability, remembering that the more nearly leveled the tripod head is kept the less the leveling screws on the instrument will have to be turned

The tripods used for the Wye and Dumpy Levels have threaded heads upon which the instruments are mounted. Tripods used for the Self-Leveling Levels are slightly different. Most have a threaded post, or rod, extending through the tripod head which is used as a means of attaching the instrument.

Care should be taken when attaching the instrument so as not to cross thread or otherwise damage the threads on either the instrument or the tripod head. The instrument should be attached to the tripod head firmly to prevent any movement during use, but it should not be attached so tightly to cause the-threads to bind. A tripod cap, which is a device to cover and protect the threads on the tripod head, should always be attached to the tripod during storage or transportation to prevent damage to the threaded areas.

After the tripod is set, the instrument is leveled by leveling screws. The Wye and Dumpy Levels have four leveling screws, and they are leveled by centering the bubble in the bubble tube, which is under, and parallel to the telescope.

Graduations are etched on the bubble tube for ease in exact centering of the bubble. To center the bubble, turn the telescope until it is aligned with a pair of opposite level screws and turn this pair of screws either both in or both



out until the bubble is approximately centered.

The screws should be gripped between the thumbs and forefingers as shown above, and uniformly turned so that the thumbs move either toward or away from each other. By turning the screws uniformly in this fashion, one screw is tightened by the same amount the other is loosened. This tilts the telescope and at the same time supports the instrument by maintaining tension on both screws. The level screws should bear firmly on the plate at all times, but should never be so tight as to bind. The inexperienced instrument person has a tendency to apply too much tension to the leveling screws.

After the telescope has been approximately leveled in the direction of one pair of level screws, the telescope should be turned at a right angle to align with the other pair of level screws, and leveled by manipulating that pair of screws as previously described. The telescope is then turned back to align with the first pair of level screws and the level bubble centered exactly; this procedure is then repeated for the second pair of screws.

Two levelings over each pair of screws should have the bubble centered exactly; however; three or more levelings may be necessary for the inexperienced instrumentman. When leveling the instrument, it is not necessary to center the bubble exactly on the first leveling. Usually to within one-half to one division is sufficient; then the bubble is centered exactly on the second leveling. If the instrument is in perfect adjustment, the bubble will remain centered as the telescope is rotated about its axis.

Bubble movement from a centered position when the telescope is aligned in a direction not parallel to a pair of screws indicates the instrument is out of adjustment. Adjustment of the level will not be covered in this text. Slight movement of the bubble, up one or two divisions, can be tolerated if the bubble is centered prior to taking the reading. In these cases, the bubble can be centered by slightly turning only one of the screws.

In setting up the Self-Leveling Level, the tripod is positioned as previously described in this section. However, the

procedure for leveling the instrument is different. Since this type of instrument relies on an optical system using a suspended prism to reflect a level line of sight, it is not necessary that the level tube be truly level when taking a reading. The Self-Leveling Level is equipped with a circular bubble vial instead of a bubble tube for use in leveling the instrument.



A right angle prism, or mirror, is usually mounted over the circular bubble vial to allow the instrument person to observe the bubble from his normal operating position, The bubble need only be inside the circle etched in the center of the circular vial for the instrument to operate satisfactorily. Thus the instrument person does not have to go through the time consuming procedure of precisely leveling the telescope tube as required for the Wye and Dumpy Levels.

The Self-Leveling Level is equipped with only three leveling screws, each one of which can be adjusted independently of the other two. When the instrument person has completed his observations and is ready to pick up the instrument he should always adjust the level screws to make the telescope approximately parallel to the foot plate of the instrument. This is easily done by adjusting the level screws until they are all exposed approximately the same amount below the level head.

Remember that the level screws should be operated as opposite pairs and be uniformly turned with the thumbs moving either toward or away from each other on the Wye and Dumpy Levels. By adjusting the level screws to make the level head parallel to the foot plate, the instrument person will make an equal range of adjustment, up or down, on the next set up. Also, it is easier to set the instrument up close to horizontal when the telescope is parallel to the tripod head and foot plate. If the foot plate and level head are not close to parallel, some instruments will not fit properly into their cases or boxes.

Reading the Rod

After the instrument is set up and leveled, the instrument person should check to make sure the cross-hairs in the telescope are focused before taking a reading. The cross-hairs are focused by moving the eyepiece in or out until the most distinct position of the cross-hairs is found. On some instruments the eyepiece may be manually moved in or out, but on most instruments the eyepiece is moved in or out by turning a knurled ring on the end of the telescope tube nearest the eyepiece.

It is best to use a light background such as the sky or light colored building when focusing the cross-hairs, and the eyepiece should be moved slowly when focusing. After this, turn the telescope toward the level rod and focus on the rod by turning the knob mounted on the top or side of the telescope tube. You will note that focusing on the rod does not affect the focus of the cross-hairs.

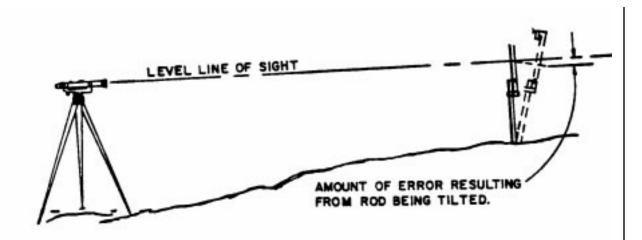
Once focused on the rod, the instrument person should check for parallax by slowly moving his eye up or down and observing whether the horizontal cross-hair appears to move on the rod. If apparent movement is observed, parallax exists, and further focusing of the cross-hairs is necessary. When apparent movement of the cross-hairs is not evident, parallax does not exist, and the cross-hairs are properly focused. Once they are properly focused, no further adjustment of the cross-hairs will be necessary as long as the same instrument person makes the reading because the eyepiece will remain in the correct position for all observations, regardless of their distance from the instrument. However, it may be necessary to re-focus the cross-hairs when another individual makes a reading. After focusing on the rod, the instrument person should check to see if the bubble is still centered.

On a self-leveling level, the bubble should be within the circle etched on the circular level vial; on either a Wye or Dumpy level the bubble should be centered in the tube. If the bubble is off by only one or two divisions, it can be centered by tightening one of the pair of opposite screws, which are more closely aligned with the telescope. If the bubble is off by more than two divisions, it would be advisable to relevel the instrument as described in the preceding section.

The rod reading is taken directly from the face of the rod, where the horizontal cross-hair appears to intersect the face of the rod. The typical level has one long horizontal cross-hair which appears to be centered in the telescope tube. This is the cross-hair used when leveling. The two shorter horizontal lines are used for stadia readings, which will not be covered in this text. The readings are normally made to the nearest one hundredth (.01); however, on occasion it may be desirable to estimate readings to the nearest five thousandths (.005). After making a reading on the Wye or Dumpy level, the instrument person should again check the bubble to be sure that it was centered during the reading.

Holding the Rod

The level rod must be held vertically for the reading to be correct; when the rod is not held plumb, the rod reading will be a greater number than the correct rod reading.



One can readily see that the amount of error increases significantly as the rod is tilted more, and as the reading is taken from nearer the top of the rod. The instrument person can tell whether the rod is vertical in the plane of the line of sight by noting if the rod is being held parallel to the vertical cross-hair, and he can instruct the rodman accordingly. However, the instrument person can not determine whether the rod is being held vertical to the direction of his line of sight.

An excellent means of keeping the rod vertical is for the rodman to balance the rod between his fingers; however, this works only when the wind is not blowing. The rodman can compare the rod to building lines, utility poles, trees, etc., to determine if it is being held plumb.

Also, rod levels are available for use as an aid in holding the rod plumb. A rod level is a device consisting of a circular level vial, or two tubular level vials mounted at right angles to each other, which may be fastened to or held on the edge of the level rod. When the bubbles are centered, the rod is plumb.

Another means of ensuring that the correct rod reading is observed is called "waving the rod". Waving the rod is waving or swinging the rod slowly back and forth toward the instrument, using the base of the rod as a pivot point. The instrument person can observe the lowest reading, which is the true vertical reading.

Determining Elevations - Leveling

An elevation has been previously defined in this text as the vertical distance above a reference or datum plane. Governmental agencies have established a network of permanent monuments with known elevations across the country. These monuments are called benchmarks. In construction surveying any point with a known elevation can be termed a benchmark. As noted earlier in this text, the Photogrammetry Unit and Location Unit normally establish a network of benchmarks along construction projects when they accumulate data for development of the plans. Definition of several common terms normally used in leveling will probably be beneficial at this point:

Benchmark (B.M.) - Monument or point with known elevation.

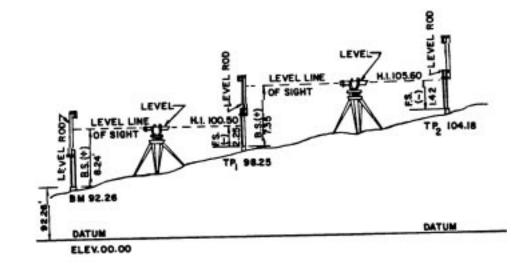
Elevation - Vertical distance above a reference or datum plane.

Backsight (B.S.) - Rod reading taken on a point of known elevation, sometimes called a plus (+) sight.

Foresight (F.S.) - Rod reading taken on a point to determine the elevation on that point, sometimes called a minus (-) sight.

Turning Point (T.P.) - An intermediate reference point used when it is necessary to move the instrument ahead while running a line of levels.

Height of Instrument (H.I.) - The elevation of the level line sight projected by the instrument, when properly set up.



The photo above illustrates a simple example of determining the elevation of a point. First, note that datum line or reference is shown. For purposes of this example, the datum is set at 0.00 elevation. The benchmark (B.M.) is

5-1. c-the vertical distance between points

5-2. b-vertical distance

5-3. Wye Dumpy Self Leveling

5-4. vertical

5-5. 4, 3, 2, 1

5-6. 786.27 + 7.30 = 793.57

shown 92.26. Remember that the elevation of a point is its vertical distance above a datum. You will also recall that a benchmark is a point with known elevation. So in reality, the elevation of the benchmark would be known prior to commencing any field work.

The level should be set up at some convenient distance from the benchmark. A good rule of thumb to follow in leveling is to keep all observations to within 300'. Observations in excess of 300' may introduce errors. The instrument person should be careful to set up the instrument at a point that is neither too high, nor too low, to read the level rod when held on the benchmark. Usually a quick check with a hand level before setting up the level will verify whether the rod can be seen from a particular point.

Once the instrument is set up, the instrument is ready to take a rod reading on the benchmark. This reading is called a backsight (B.S.) or a plus (+) shot. From the photo above, assume the rod reading on the B.S. is 8.24, one can readily see that by adding (+) the B.S. (8.24) to the B.M. elevation (92.26), the height of instrument (H.I.) is calculated to be 92.26 + 8.24 or 100.50. Remember B.S., or plus shots, are always added.

Once the height of instrument is known, the elevation of other points can be determined from that set up as long as a rod reading is visible on the point. If the instrument person is unable to observe a rod reading on the point due to difference in elevation, distance limitations, physical obstructions, etc., it will be necessary to establish one or more turning points (T.P.) in order to progress to the required points.

A turning point should be a solid, clearly defined point, such as a stake or embedded rock, which is not likely to change in elevation during the leveling process. It should also be of such configuration that the elevation of the bottom of the rod will not change when the rod is faced in different directions.

To determine the elevation of a point a rod reading is taken on that point. This reading is called a frontshot (F.S.) or minus (-) shot. The elevation of the point is then determined by subtracting the F.S. from the H.I. As in the photo, the F.S. (2.25) is subtracted from the H.I. (100.50) to determine the elevation of T.P.1 (100.50- 2.25) = 98.25. Remember frontshots are rod readings taken to determine the elevations of points; frontshots are always subtracted from the H.I. to calculate elevations.

Using the principles above, one can readily calculate the elevation of T.P. 2 in the above photo. Once the elevation of T.P.1 is determined, the level is then moved ahead and set up. Since the elevation of T.P.1 is known, the instrument person can take a B.S. on T.P.1, calculate the H.I. for the set up, take an F.S. on T.P.2, and calculate the elevation of T.P.2. Elevation of T.P.1 (98.25') + B.S. (7.35') = H.I. (105.60'). Then the H.I. (105.60') - F.S. (1.42') = the elevation of T.P.2 (104.18'). Using this technique one can progress through as many turning points as necessary to reach a particular point.

Whenever a line of levels is run involving two or more set ups, the field work should always be checked by running a line of levels back to the original B.M. or to another B.M. which has been previously established from

the same datum. This will detect significant errors in the observations.

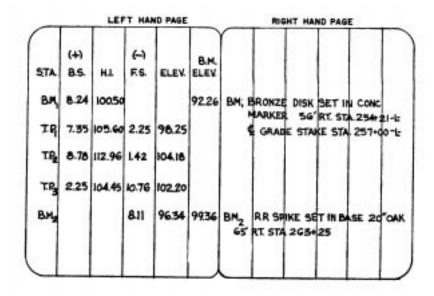
Answer the questions.

5-1.						
a. the horizontal distance between points						
b. the distance the spreader and roller						
	c. the vertical	distance between points				
	d. the distance	between the stake and hub				
5-2.		above a reference plane, or datum.				
	a. horizontal d	istance				
	b. vertical dist	ance				
	c. corrected po	bint				
	d. staking part					
5-3.		els used in construction surveying.				
	• •					
	2					
	2					
	5					
5-4.	A banchmark is the refe	rence normally used to establish (horizontal/vertical) control on highway				
5-4.		Tence normany used to establish (norizontal/vertical) control on highway				
	construction projects.					
5-5.	Matali the fellowing cal					
5-5.	Match the following col	iumns:				
C	olumn A	Column B				
	a. Backsight	1. The elevation of the level line sight projected by the instrument, when properly set up.				
	b. Foresight	2. An intermediate reference point used when it is necessary to move the instrument ahead while running a line of levels.				
	c. Turning Point	3. Rod reading taken on a point to determine the elevation on that point, sometimes called a minus (-) sight.				
	d. H.I.	4.Rod reading taken on a point of known elevation, sometimes called a plus (+) sight.				
5-6.	If the B.M. is at 786.27	above sea level and the Backsight is 7.30, what is the H. I.?				

- **5-7.** If the F.S. reads 1.98, what is the elevation of the First Turning Point (T.P.1)? Use the H.I. in the above example.
- **5-8.** What is the elevation of T.P.2, if the B.S. is 4.56 and the F.S. is 2.44?
- **5-9.** The rodperson must hold the level rod ______ when it is being read.
 - a. diagonally
 - b. as best they can
 - c. vertically
 - d. horizontally

CHAPTER 6: NOTEKEEPING

Obviously level notes cannot be kept on sketches such as the photo above. In construction surveying, level notes are kept in bound level books furnished by the Department. As with all field notes, level notes should be written with a 3H pencil or harder, and with no erasures. Erroneous entries should be lined through and initialed, not erased. Also, for each day's operation, the date, party members and their respective duties, and the weather should be recorded. Level notes are normally kept in a fashion as shown below:



Some survey parties may use slight variations of the above example; however, the format used for the first four column headings is almost universal. The right hand page is normally used exclusively for descriptive notes and comments. B.M.'s should always be described in the notes. A description of the T.P.'s is optional; however, if some permanent feature is used as a T.P., it may be helpful to record a description since it could possibly be of value later.

Level notes are generally recorded starting at the top of the page and running down the page. Field data (rod readings, elevations, etc.) is usually recorded on the left-hand sheet. The rod readings (both F.S. and B.S.) are recorded on the same horizontal line for a particular point. However, the F.S. is subtracted from the previous H.I. to calculate the elevation of that particular point. The B.S. is then added to the elevation to calculate the H.I., and the process is then repeated as necessary to reach a given point.

- 6-1. a-in bound level books
- 6-2. c-lined through and initialed

It may be helpful to refer to the elevation photo when reviewing the data recorded in the photo above. Note that elevation computed for B.M.2 is 0.02' off the given B.M.2 elevation. This amount of error may or may not be acceptable; the degree of accuracy required is dependent upon the type of work involved.

Referring back to the elevation photo, it can be seen that the difference in elevation between any two points is equal to the difference in the sum of the backshots and the sum of the frontshots between those two points. For example, between B.M. and T.P.2, the sum of B.S. is 15.59' and the sum of F.S. is 3.67'. Since backshots are plus (+) and frontshots are minus (-), the difference in the sums is +11.92'. By adding this figure algebraically to the B.M. elevation a figure of 104.18' is computed. This figure is the same as the elevation calculated for T.P.2, thus a mathematical check of the computed notes is provided.

Now apply this same check to the notes. Between B.M. and B.M.2 the sum of backshots is +26.62' and the sum of the frontshots is -22.54'; the difference in the two sums is 4.08'. This figure added algebraically to the elevation of B.M. is 92.26' + 4.08' or 96.34', the computed elevation of B.M.2. Thus the calculated notes are mathematically correct. This method provides a quick check of mathematics, which can be easily done in the field, and it is recommended that some type of check be made on field notes.

When running profile levels, normally several observations are made from the same H. I. Consequently, the notes will have to be recorded slightly different. Rod readings are usually read and recorded to the nearest tenth of a foot (0.1') on earth material, and to the nearest one hundredth (0.01') on pavements, turning points, benchmarks, or other permanent features. As previously stated, the degree of accuracy is dependent upon the type of work involved. For profile levels, notes are usually recorded in a manner similar to the following photo.

-	-	EFT HA	AND PA	GE		RIGHT HAND PAGE
STA.	(+) BS	н.	(-) F.S.	ELEV.	B.H.	
BM,	824	100.50			92.26	BM BRONZE DISK SET IN CONCRETE
255-00			7.4	95.1		MARKER 56'RT 254+21-L
+50			5.)	95.4		
256400			3.7	96.8		
тр	7.85	105.60	2.25	98.25		
256+50			7.10	98.5		
257-00			6.20	99.4		

5-7. 793.57 - 1.98 = 791.59 = T.P. 1

5-8.

791.59 + 4.56 = 796.15 = H.I. 796.15 - 2.44 = 793.71 = T.P. 2

5-9. c-vertically

Up to this point the text has discussed leveling procedures using the Philadelphia type level rod only.

Answer the questions.

- 6-1. Level notes are kept _____
 - a. in bound level books
 - b. in the refrigerator
 - c. in stenographer pads
 - d. in the project manager's diary

6-2. Erroneous entries in level notes should be

- a. crossed out and covered with correction fluid
- b. erased and rewritten
- c. lined through and initialed
- d. washed down and dried out

CHAPTER 7: COMMON SOURCES OF ERRORS AND MISTAKES IN LEVELING

The main requirement of any level is for the line of sight to be projected parallel to the level tube (excluding selfleveling types) and for the line of sight to be projected perpendicular to the vertical axis. The amount of error resulting from an improper adjustment increases in direct proportion to the distance from the rod to the instrument

Instrument out of Adjustment

If an instrument is out of adjustment, and the line of sight is not truly level, then the error can be eliminated or minimized by taking backsights and frontsights at approximately equal distances from the instrument. The distances may be obtained by pacing; or, in the case of a line of levels, the sum of the distances to the backsights should approximately equal the sum of the distances to the frontsights. This is a very important principle to observe in leveling, and it should always be adhered to where practical. The distances to backsights should equal the distances to frontsights.

Bubble Not Centered When Taking Rod Reading

This applies to Wye and Dumpy type levels. The instrument person should always check the bubble when making a rod reading.

Rod Not Held Plumb

The rodman can "balance" the rod between his fingers if the wind is not blowing. Rod levels may be used; the rodman can use the horizon, building lines, trees, or other references to hold the rod plumb; and the instrument person can use the vertical hairline to assure that the rod is plumb in one direction. "Waving the rod," as discussed earlier in the text, is an effective means of eliminating this type of error, particularly when reading high on the rod.

Turning Points

Turning points should be some solid, well defined point that is not apt to change in elevation while the instrument person moves ahead. A marked spot on pavement, a stake firmly set, an embedded rock are some examples of items which can be used as turning points. It is very important that the frontsight and backsight be taken on exactly the same spot on a T.P.

Erroneous Rod Length

New rods should be checked with a steel tape to verify that the face is properly graduated. However, the most common error relating to rod length is failure to clamp the top sections of the rod at the proper place when using

	the "high rod." The rodman should always extend the rod fully before clamping it, and he should periodically check to see that the rod does not slip from its fully extended position when using the high rod.
	Parallax - Improper Focusing The check for parallax was discussed earlier in the text, as was the procedure for eliminating parallax. When parallax exists, incorrect rod readings are likely to occur. The instrument person should check for parallax each day, and each time it is necessary to focus the cross-hairs.
Summary a. 6.25"	Other Careless Mistakes and Errors Some others of the more frequent careless mistakes include: reading the wrong foot-mark; reading the wrong tenth-mark; entering the F.S. in the B.S. column of the field book, or vice versa; adding the F.S. or subtracting the B.S. when computing the notes. Concentration on individual job tasks is probably the most effective means of reducing careless mistakes.
b. H.I. = Elevation at Point A + BS = 10/00" + 6.25"	Care of Equipment As with any other precision measuring equipment, the level should be carefully handled. When being transported in a vehicle, the instrument should be in its box or case. Level boxes or carrying cases are fitted to accept the instrument and afford protection while it is being transported.
c. 2.52'	The instrument should be handled by the base plate, not the telescope, when transferring it from the tripod to the box or vice versa. In the field, the instrument is normally carried over the shoulder with the tripod legs pointing forward; however, in close quarters, thickly wooded areas, entering doorways, etc., the instrument head should be held forward to avoid striking it on objects. Special care should be exercised to avoid striking or jolting the instrument, as this can result in the instrument being out of adjustment.
d. Elev. at Pt. B = H.I F.S. = 106.25 - 2.52 Elevation at Point B = 103.73	The instrument should not be allowed to get wet, but in case of a quick rain shower, all metal parts should be wiped dry before storing the instrument. When cleaning the telescope lens, care should be exercised to avoid scratching the glass. The glass should be first brushed with a camel's hair brush, then gently wiped with an old soft piece of linen moistened with alcohol to dissolve any grease on the lens, and finally brushed again with a camel's hair brush. Special lens paper is also available for cleaning the lens.
	The level rod should always be stored with the clamps loose. These rods are made of wood and are affected by moisture; and if stored with the clamps tight, swelling may occur making it difficult to release the clamps. The level should be used and stored in such a manner as to prevent damage to the graduated face.
12	

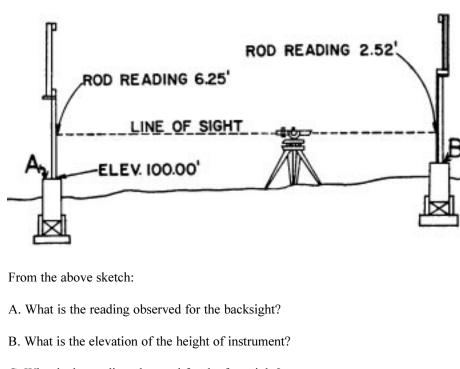
7-1.	Name five (5) common sources of errors and mistakes in leveling.				
7-2.	A rod can be plumb.	between his fingers if the wind is not blowing to hold it			
	a. twirled				
	b. balanced				
	c. flipped				

d. laid flat

SUMMARY

After studying this text, the student should have a basic understanding of the measurement of horizontal distances (chaining), and the measurement of vertical distances (leveling). The only way to become proficient in making these measurements is through experience, and hopefully by observing the principles set forth in this text, the student will become proficient at a quicker pace.

The serious student would do well to secure a textbook on surveying for more detailed study. Surveying, by Charles B. Breed, is a excellent book which is available at most college bookstores.



Answer the questions.

- C. What is the reading observed for the frontsight?
- D. What is the elevation of Point B?

7-1.

instrument out of adjustment

• bubble not centered when taking rod reading

•rod not held plumb

• turning point not solid

• erroneous rod length

• parallax - improper focus

• reading rod wrong

• entering data in the wrong column

• adding or subtracting the wrong number

7-2. b-balanced